Correction : coordonnées d'un point dans une égalité de vecteurs

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✎ Exercice 1

$\boxed{A(-2\ ;\ 3),\ B(-3\ ;\ -1),\ C(0\ ;\ 2) \text{ et } D(0\ ;\ 4)}$.

1) $\boxed{\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{CD}}$:

D'une part, $\overrightarrow{AM}\begin{pmatrix} x_M - x_A \\ y_M - y_A \end{pmatrix}$; $\boxed{\overrightarrow{AM}\begin{pmatrix} x_M + 2 \\ y_M - 3 \end{pmatrix}}$.

D'autre part, $\overrightarrow{AB}\begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$; $\overrightarrow{AB}\begin{pmatrix} -3+2 \\ -1-3 \end{pmatrix}$; $\overrightarrow{AB}\begin{pmatrix} -1 \\ -4 \end{pmatrix}$; et $\overrightarrow{CD}\begin{pmatrix} x_D - x_C \\ y_D - y_C \end{pmatrix}$; $\overrightarrow{CD}\begin{pmatrix} 0-0 \\ 4-2 \end{pmatrix}$; $\overrightarrow{CD}\begin{pmatrix} 0 \\ 2 \end{pmatrix}$.

Donc $\overrightarrow{AB} + \overrightarrow{CD}\begin{pmatrix} -1+0 \\ -4+2 \end{pmatrix}$; $\boxed{\overrightarrow{AB} + \overrightarrow{CD}\begin{pmatrix} -1 \\ -2 \end{pmatrix}}$.

$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{CD} \iff \begin{cases} x_M + 2 = -1 \\ y_M - 3 = -2 \end{cases} \iff \begin{cases} x_M = -1-2 \\ y_M = -2+3 \end{cases} \iff \begin{cases} x_M = -3 \\ y_M = 1 \end{cases}$. Donc $\boxed{M(-3\ ;\ 1)}$.

2) $\boxed{\overrightarrow{BK} = 3\overrightarrow{AC} - 2\overrightarrow{BA} = 3\overrightarrow{AC} + 2\overrightarrow{AB}}$:

D'une part, $\overrightarrow{BK}\begin{pmatrix} x_K - x_B \\ y_K - y_B \end{pmatrix}$; $\boxed{\overrightarrow{BK}\begin{pmatrix} x_K + 3 \\ y_K + 1 \end{pmatrix}}$.

D'autre part, $\overrightarrow{AC}\begin{pmatrix} x_C - x_A \\ y_C - y_A \end{pmatrix}$; $\overrightarrow{AC}\begin{pmatrix} 0+2 \\ 2-3 \end{pmatrix}$; $\overrightarrow{AC}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$; donc $3\overrightarrow{AC}\begin{pmatrix} 3\times 2 \\ 3\times(-1) \end{pmatrix}$; $3\overrightarrow{AC}\begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

$\overrightarrow{AB}\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ d'après la question précédente, donc $2\overrightarrow{AB}\begin{pmatrix} 2\times(-1) \\ 2\times(-4) \end{pmatrix}$; $2\overrightarrow{AB}\begin{pmatrix} -2 \\ -8 \end{pmatrix}$.

Ainsi $3\overrightarrow{AC} + 2\overrightarrow{AB}\begin{pmatrix} 6-2 \\ -3-8 \end{pmatrix}$; $\boxed{3\overrightarrow{AC} + 2\overrightarrow{AB}\begin{pmatrix} 4 \\ -11 \end{pmatrix}}$.

$\overrightarrow{BK} = 3\overrightarrow{AC} + 2\overrightarrow{AB} \iff \begin{cases} x_K + 3 = 4 \\ y_K + 1 = -11 \end{cases} \iff \begin{cases} x_K = 4-3 \\ y_K = -11-1 \end{cases} \iff \begin{cases} x_K = 1 \\ y_K = -12 \end{cases}$. Donc $\boxed{K(1\ ;\ -12)}$.

✎ Exercice 2

$\boxed{A(-2\ ;\ 5),\ B(2\ ;\ -2) \text{ et } C(6\ ;\ 3)}$.

1) $\boxed{\overrightarrow{AE} = \frac{3}{2}\overrightarrow{AC}}$:

D'une part, $\overrightarrow{AE}\begin{pmatrix} x_E - x_A \\ y_E - y_A \end{pmatrix}$; $\boxed{\overrightarrow{AE}\begin{pmatrix} x_E + 2 \\ y_E - 5 \end{pmatrix}}$.

D'autre part, $\overrightarrow{AC}\begin{pmatrix} x_C - x_A \\ y_C - y_A \end{pmatrix}$; $\overrightarrow{AC}\begin{pmatrix} 6+2 \\ 3-5 \end{pmatrix}$; $\overrightarrow{AC}\begin{pmatrix} 8 \\ -2 \end{pmatrix}$; donc $\frac{3}{2}\overrightarrow{AC}\begin{pmatrix} \frac{3}{2}\times 8 \\ \frac{3}{2}\times(-2) \end{pmatrix}$; $\boxed{\frac{3}{2}\overrightarrow{AC}\begin{pmatrix} 12 \\ -3 \end{pmatrix}}$.

$\overrightarrow{AE} = \frac{3}{2}\overrightarrow{AC} \iff \begin{cases} x_E + 2 = 12 \\ y_E - 5 = -3 \end{cases} \iff \begin{cases} x_E = 12-2 \\ y_E = -3+5 \end{cases} \iff \begin{cases} x_E = 10 \\ y_E = 2 \end{cases}$. Donc $\boxed{E(10\ ;\ 2)}$.

2) $\boxed{\overrightarrow{BF} = -\frac{1}{8}\overrightarrow{BA} + \frac{3}{8}\overrightarrow{BC} = \frac{1}{8}\overrightarrow{AB} + \frac{3}{8}\overrightarrow{BC}}$:

D'une part, $\overrightarrow{BF}\begin{pmatrix} x_F - x_B \\ y_F - y_B \end{pmatrix}$; $\boxed{\overrightarrow{BF}\begin{pmatrix} x_F - 2 \\ y_F + 2 \end{pmatrix}}$.

D'autre part, $\overrightarrow{AB}\begin{pmatrix} x_B - x_A \\ y_B - y_A \end{pmatrix}$; $\overrightarrow{AB}\begin{pmatrix} 2+2 \\ -2-5 \end{pmatrix}$; $\overrightarrow{AB}\begin{pmatrix} 4 \\ -7 \end{pmatrix}$; donc $\frac{1}{8}\overrightarrow{AB}\begin{pmatrix} \frac{1}{8}\times 4 \\ \frac{1}{8}\times(-7) \end{pmatrix}$; $\frac{1}{8}\overrightarrow{AB}\begin{pmatrix} \frac{1}{2} \\ -\frac{7}{8} \end{pmatrix}$.

$\overrightarrow{BC}\begin{pmatrix}x_C-x_B\\y_C-y_B\end{pmatrix}$; $\overrightarrow{BC}\begin{pmatrix}6-2\\3+2\end{pmatrix}$; $\overrightarrow{BC}\begin{pmatrix}4\\5\end{pmatrix}$; donc $\frac{3}{8}\overrightarrow{BC}\begin{pmatrix}\frac{3}{8}\times 4\\\frac{3}{8}\times 5\end{pmatrix}$; $\frac{3}{8}\overrightarrow{BC}\begin{pmatrix}\frac{3}{2}\\\frac{15}{8}\end{pmatrix}$.

Ainsi $\frac{1}{8}\overrightarrow{AB}+\frac{3}{8}\overrightarrow{BC}\begin{pmatrix}\frac{1}{2}+\frac{3}{2}\\-\frac{7}{8}+\frac{15}{8}\end{pmatrix}$; $\frac{1}{8}\overrightarrow{AB}+\frac{3}{8}\overrightarrow{BC}\begin{pmatrix}\frac{4}{2}\\\frac{8}{8}\end{pmatrix}$; $\boxed{\frac{1}{8}\overrightarrow{AB}+\frac{3}{8}\overrightarrow{BC}\begin{pmatrix}2\\1\end{pmatrix}}$.

$\overrightarrow{BF}=\frac{1}{8}\overrightarrow{AB}+\frac{3}{8}\overrightarrow{BC} \iff \begin{cases}x_F-2=2\\y_F+2=1\end{cases} \iff \begin{cases}x_F=2+2\\y_F=1-2\end{cases} \iff \begin{cases}x_F=4\\y_F=-1\end{cases}$. Donc $\boxed{F(4\ ;\ -1)}$.

✎ Exercice 3

$\boxed{I(-1\ ;\ 3) \text{ et } J(1\ ;\ -2)}$.

$\boxed{\overrightarrow{MI}+2\overrightarrow{MJ}=\vec{0}}$:

D'une part, $\overrightarrow{MI}\begin{pmatrix}x_I-x_M\\y_I-y_M\end{pmatrix}$; $\overrightarrow{MI}\begin{pmatrix}-1-x_M\\3-y_M\end{pmatrix}$.

$\overrightarrow{MJ}\begin{pmatrix}x_J-x_M\\y_J-y_M\end{pmatrix}$; $\overrightarrow{MJ}\begin{pmatrix}1-x_M\\-2-y_M\end{pmatrix}$; donc $2\overrightarrow{MJ}\begin{pmatrix}2(1-x_M)\\2(-2-y_M)\end{pmatrix}$; $2\overrightarrow{MJ}\begin{pmatrix}2-2x_M\\-4-2y_M\end{pmatrix}$.

Donc $\overrightarrow{MI}+2\overrightarrow{MJ}\begin{pmatrix}-1-x_M+2-2x_M\\3-y_M-4-2y_M\end{pmatrix}$; $\boxed{\overrightarrow{MI}+2\overrightarrow{MJ}\begin{pmatrix}1-3x_M\\-1-3y_M\end{pmatrix}}$.

D'autre part, $\boxed{\vec{0}\begin{pmatrix}0\\0\end{pmatrix}}$.

$\overrightarrow{MI}+2\overrightarrow{MJ}=\vec{0} \iff \begin{cases}1-3x_M=0\\-1-3y_M=0\end{cases} \iff \begin{cases}1=3x_M\\-1=3y_M\end{cases} \iff \begin{cases}x_M=\frac{1}{3}\\y_M=-\frac{1}{3}\end{cases}$. Donc $\boxed{M\left(\frac{1}{3}\ ;\ -\frac{1}{3}\right)}$.

✎ Exercice 4

$\boxed{A(-2\ ;\ 1) \text{ et } B(3\ ;\ 3)}$.

$\boxed{2\overrightarrow{PA}+3\overrightarrow{PB}=\overrightarrow{AB}}$:

D'une part, $\overrightarrow{PA}\begin{pmatrix}x_A-x_P\\y_A-y_P\end{pmatrix}$; $\overrightarrow{PA}\begin{pmatrix}-2-x_P\\1-y_P\end{pmatrix}$; donc $2\overrightarrow{PA}\begin{pmatrix}2(-2-x_P)\\2(1-y_P)\end{pmatrix}$; $2\overrightarrow{PA}\begin{pmatrix}-4-2x_P\\2-2y_P\end{pmatrix}$.

$\overrightarrow{PB}\begin{pmatrix}x_B-x_P\\y_B-y_P\end{pmatrix}$; $\overrightarrow{PB}\begin{pmatrix}3-x_P\\3-y_P\end{pmatrix}$; donc $3\overrightarrow{PB}\begin{pmatrix}3(3-x_P)\\3(3-y_P)\end{pmatrix}$; $3\overrightarrow{PB}\begin{pmatrix}9-3x_P\\9-3y_P\end{pmatrix}$.

Donc $2\overrightarrow{PA}+3\overrightarrow{PB}\begin{pmatrix}-4-2x_P+9-3x_P\\2-2y_P+9-3y_P\end{pmatrix}$; $\boxed{2\overrightarrow{PA}+3\overrightarrow{PB}\begin{pmatrix}5-5x_P\\11-5y_P\end{pmatrix}}$.

D'autre part, $\overrightarrow{AB}\begin{pmatrix}x_B-x_A\\y_B-y_A\end{pmatrix}$; $\overrightarrow{AB}\begin{pmatrix}3+2\\3-1\end{pmatrix}$; $\boxed{\overrightarrow{AB}\begin{pmatrix}5\\2\end{pmatrix}}$.

$2\overrightarrow{PA}+3\overrightarrow{PB}=\overrightarrow{AB} \iff \begin{cases}5-5x_P=5\\11-5y_P=2\end{cases} \iff \begin{cases}5-5=5x_P\\11-2=5y_P\end{cases} \iff \begin{cases}0=5x_P\\9=5y_P\end{cases} \iff \begin{cases}x_P=\frac{0}{5}\\y_P=\frac{9}{5}\end{cases} \iff \begin{cases}x_P=0\\y_P=\frac{9}{5}\end{cases}$.

Donc $\boxed{P\left(0\ ;\ \frac{9}{5}\right)}$.