Correction : résoudre une équation dans $\mathbb{C}$

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✎ Exercice

a) $2z-5=10-iz \iff 2z+iz=10+5 \iff (2+i)z=15 \iff z=\dfrac{15}{2+i} \iff z=\dfrac{15(2-i)}{2^2+1^2}$
$\iff z=\dfrac{30-15i}{4+1} \iff z=\dfrac{30-15i}{5} \iff z=6-3i$. La solution de l'équation est $\boxed{z=6-3i}$.

b) $2\overline{z}=i-7 \iff \overline{z}=\dfrac{i-7}{2} \iff \overline{z}=\dfrac{-7+i}{2} \iff z=\dfrac{-7-i}{2}$. La solution de l'équation est $\boxed{z=\dfrac{-7-i}{2}}$.

c) $\dfrac{\overline{z}-1}{\overline{z}+1}=i$. Valeur interdite : $\overline{z}+1=0 \iff \overline{z}=-1 \iff z=-1$.
$\dfrac{\overline{z}-1}{\overline{z}+1}=i \iff \overline{z}-1=i(\overline{z}+1) \iff \overline{z}-1=i\overline{z}+i \iff (1-i)\overline{z}=i+1 \iff \overline{z}=\dfrac{i+1}{1-i} \iff \overline{z}=\dfrac{(i+1)(1+i)}{1^2+1^2}$
$\iff \overline{z}=\dfrac{(1+i)^2}{2} \iff \overline{z}=\dfrac{1+2i+i^2}{1+1} \iff \overline{z}=\dfrac{\cancel{1}+2i-\cancel{1}}{2} \iff \overline{z}=\dfrac{2i}{2} \iff \overline{z}=i \iff z=-i$.
Or $-i\neq -1$ donc la solution de l'équation est $\boxed{z=-i}$.

d) $z=3\overline{z}+2-6i$. Posons $z=x+iy$ avec $x\in\mathbf{R}$ et $y\in\mathbf{R}$. Ainsi $\overline{z}=x-iy$.
$z=3\overline{z}+2-6i \iff x+iy=3(x-iy)+2-6i \iff x+iy=3x-3iy+2-6i \iff x+iy=(3x+2)-i(3y+6)$
$\iff \begin{cases} x=3x+2 \\ y=-3y-6 \end{cases} \iff \begin{cases} -2x=2 \\ 4y=-6 \end{cases} \iff \begin{cases} x=\dfrac{2}{-2} \\ y=\dfrac{-6}{4} \end{cases} \iff \begin{cases} x=-1 \\ y=-\dfrac{3}{2} \end{cases} \iff z=-1-\dfrac{3}{2}i.$

La solution de l'équation est $\boxed{z=-1-\dfrac{3}{2}i}$.

e) $(2-i)z-4+i=0 \iff (2-i)z=4-i \iff z=\dfrac{4-i}{2-i} \iff z=\dfrac{(4-i)(2+i)}{2^2+1^2} \iff z=\dfrac{8+4i-2i-i^2}{4+1}$
$\iff z=\dfrac{8+2i+1}{5} \iff z=\dfrac{9+2i}{5}$. La solution de l'équation est $\boxed{z=\dfrac{9+2i}{5}}$.

f) $2iz-3+4i=(1-3i)z+5 \iff -3+4i-5=(1-3i)z-2iz \iff -8+4i=(1-3i-2i)z \iff -8+4i=(1-5i)z$
$\iff (1-5i)z=-8+4i \iff z=\dfrac{-8+4i}{1-5i} \iff z=\dfrac{(-8+4i)(1+5i)}{1^2+5^2} \iff z=\dfrac{-8-40i+4i+20i^2}{1+25} \iff z=\dfrac{-8-36i-20}{26}$
$\iff z=\dfrac{-28-36i}{26} \iff z=\dfrac{-14-18i}{13}$. La solution de l'équation est $\boxed{z=\dfrac{-14-18i}{13}}$.

g) $\dfrac{1-i}{z+2}=i$. Valeur interdite : $z+2=0 \iff z=-2$.
$\dfrac{1-i}{z+2}=i \iff 1-i=i(z+2) \iff 1-i=iz+2i \iff 1-i-2i=iz \iff iz=1-3i \iff z=\dfrac{1-3i}{i}$
$\iff z=\dfrac{(1-3i)\times(-i)}{0^2+1^2} \iff z=\dfrac{-i+3i^2}{1} \iff z=-i-3 \iff z=-3-i$.
Or $-3-i\neq -2$ donc la solution de l'équation est $\boxed{z=-3-i}$.

h) $(2+3i)z=4i\overline{z}-3i$. Posons $z=x+iy$ avec $x\in\mathbf{R}$ et $y\in\mathbf{R}$. Ainsi $\overline{z}=x-iy$.
$(2+3i)z=4i\overline{z}-3i \iff (2+3i)(x+iy)=4i(x-iy)-3i \iff 2x+2iy+3ix+3i^2y=4ix-4i^2y-3i$
$\iff 2x-3y+i(3x+2y)=4y+i(4x-3) \iff \begin{cases} 2x-3y=4y \\ 3x+2y=4x-3 \end{cases} \iff \begin{cases} 2x=7y \\ 2y+3=x \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ 2y+3=x \end{cases}$
$\iff \begin{cases} y=\dfrac{2}{7}x \\ 2\times\dfrac{2}{7}x+3=x \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ \dfrac{4}{7}x+3=x \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ x-\dfrac{4}{7}x=3 \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ \dfrac{7}{7}x-\dfrac{4}{7}x=3 \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ \dfrac{3}{7}x=3 \end{cases}$
$\iff \begin{cases} y=\dfrac{2}{7}x \\ x=\cancel{3}\times\dfrac{7}{\cancel{3}} \end{cases} \iff \begin{cases} y=\dfrac{2}{7}x \\ x=7 \end{cases} \iff \begin{cases} y=\dfrac{2}{\cancel{7}}\times\cancel{7} \\ x=7 \end{cases} \iff \begin{cases} y=2 \\ x=7 \end{cases} \iff z=7+2i.$
La solution de l'équation est $\boxed{z=7+2i}$.

i) $iz - 3\overline{z} = 1 - i\overline{z}$. Posons $z = x + iy$ avec $x \in \mathbf{R}$ et $y \in \mathbf{R}$. Ainsi $\overline{z} = x - iy$.

$iz - 3\overline{z} = 1 - i\overline{z} \iff i(x+iy) - 3(x-iy) = 1 - i(x-iy) \iff ix + i^2y - 3x + 3iy = 1 - ix + i^2y$

$\iff -3x - y + i(x+3y) = 1 - y - ix \iff \begin{cases} -3x - y = 1 - y \\ x + 3y = -x \end{cases} \iff \begin{cases} -3x = 1 - \cancel{y} + \cancel{y} \\ x + x = -3y \end{cases} \iff \begin{cases} -3x = 1 \\ 2x = -3y \end{cases}$

$\iff \begin{cases} x = -\dfrac{1}{3} \\ y = -\dfrac{2}{3}x \end{cases} \iff \begin{cases} x = -\dfrac{1}{3} \\ y = -\dfrac{2}{3} \times \left(-\dfrac{1}{3}\right) \end{cases} \iff \begin{cases} x = -\dfrac{1}{3} \\ y = \dfrac{2}{9} \end{cases} \iff z = -\dfrac{1}{3} + \dfrac{2}{9}i.$

La solution de l'équation est $\boxed{z = -\dfrac{1}{3} + \dfrac{2}{9}i}$.