Correction : les nombres complexes sous forme algébrique

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✎ Exercice 1

$\boxed{z = 4-3i \text{ et } z' = 2+i}$.

1) $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$ donc $|z| = \sqrt{25} = \boxed{5}$.

$|z'|^2 = 2^2 + 1^2 = 4 + 1 = 5$ donc $|z'| = \boxed{\sqrt{5}}$.

2) **a)** $z + z' = 4 - 3i + 2 + i = \boxed{6-2i}$.

b) $z - z' = 4 - 3i - (2+i) = 4 - 3i - 2 - i = \boxed{2-4i}$.

c) $\dfrac{1}{z} = \dfrac{1}{4-3i} = \dfrac{4+3i}{4^2+3^2} = \dfrac{4+3i}{16+9} = \boxed{\dfrac{4+3i}{25}}$.

d) $\dfrac{z}{z'} = \dfrac{4-3i}{2+i} = \dfrac{(4-3i)(2-i)}{2^2+1^2} = \dfrac{8-4i-6i+3i^2}{4+1} = \dfrac{8-10i+3\times(-1)}{5} = \dfrac{8-3-10i}{5} = \dfrac{5-10i}{5} = \dfrac{\cancel{5}(1-2i)}{\cancel{5}}$

$\dfrac{z}{z'} = \boxed{1-2i}$.

✎ Exercice 2

$\boxed{z = -2-i \text{ et } z' = -4+6i}$.

1) $|z|^2 = (-2)^2 + (-1)^2 = 4 + 1 = 5$ donc $|z| = \boxed{\sqrt{5}}$.

$|z'|^2 = (-4)^2 + 6^2 = 16 + 36 = 52$ donc $|z'| = \sqrt{52} = \sqrt{4\times 13} = \boxed{2\sqrt{13}}$.

2) **a)** $zz' = (-2-i)(-4+6i) = 8 - 12i + 4i - 6i^2 = 8 - 8i - 6\times(-1) = 8 + 6 - 8i = \boxed{14-8i}$.

b) $z^2 = (-2-i)^2 = (-2)^2 - 2\times(-2)\times i + i^2 = 4 + 4i - 1 = \boxed{3+4i}$.

c) $\dfrac{1}{z'} = \dfrac{1}{-4+6i} = \dfrac{-4-6i}{(-4)^2+6^2} = \dfrac{-4-6i}{16+36} = \dfrac{-4-6i}{52} = \dfrac{\cancel{2}(-2-3i)}{\cancel{2}\times 26} = \boxed{\dfrac{-2-3i}{26}}$.

d) $\dfrac{z'}{z} = \dfrac{-4+6i}{-2-i} = \dfrac{(-4+6i)(-2+i)}{(-2)^2+1^2} = \dfrac{8-4i-12i+6i^2}{4+1} = \dfrac{8-16i+6\times(-1)}{5} = \dfrac{8-6-16i}{5} = \boxed{\dfrac{2-16i}{5}}$.